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INFLUENCE OF ADDENDUM MODIFICATIONS ON DYNAMIC FRICTION LOADS OF SPUR GEARS

BY

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Abstract. The paper presents an analytical investigation on the dynamic friction loads of spur gear pairs. The methodology to evaluate friction load considers the shared normal load and instantaneous friction coefficient of the contacting teeth. The time varying mesh stiffness is incorporated into dynamic model. A comparative study has been performed to investigate the effects of the addendum modification coefficients on the time-varying sliding friction between contacting teeth and on the dynamic friction loads of gear systems.

Key words: gears, friction coefficient, dynamic friction load.

1. Introduction

Friction in gear systems is considered as a dynamic excitation source in the models of these mechanisms (Velex, 2002; Gunda, 2003). Some advanced dynamic models are based on the assumption that the load is equally shared among the tooth pairs in contact (Vaishya & Singh, 2003). At the same time, the Coulomb friction is assumed with a constant coefficient of friction. These assumptions not lead to a realistic dynamic model for the friction loads of gear pairs.

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Experimental results indicated that the coefficient of friction does not remain constant during a meshing cycle (Radzimovsky, 1975; Rebbechi, 1996). The instantaneous friction coefficient is difficult to accurately calculate. Martin (1978) has provided an extensive review of the existing equations for the coefficient of friction in gearing, but the numerical results are not in good agreement with the experimental ones.

An alternative calculus methodology for the instantaneous friction coefficient of spur gear pairs is considered in the paper. The frictional forces on the gear tooth surfaces are computed by considering instantaneous friction coefficient and the sharing normal load of the meshing cycle in relation to the variable mesh stiffness (Atanasiu, 2008).

The present paper will enhance the previous works (Atanasiu, 2008; Atanasiu, 2009) by including the effect of the addendum modifications on the tooth friction loads of spur gears under dynamic conditions.

2. Spur Gear Dynamic Model

Fig. 1 presents the distribution of the dynamic normal loads F_n^i and friction loads F_f^i among the pairs of teeth in contact, where $i = I, II$ represents the pair of contacting teeth *I* and *II*, respectively. In this figure, the following mesh points were used to represent the successive positions of contact point of a tooth as it passes through the zone of loading: the initial point of engagement, A; the lowest point of single-tooth contact, B; the highest point of single-tooth contact, D; and the final point of engagement, E.

The differential equations of motion can be expressed as:

$$J_1\ddot{\theta}_1 = T_1 - \sum_{i=I}^{II} r_{b1}F_n^i + \sum_{i=I}^{II} l_1^i F_f^i \tag{1}$$

$$J_2\ddot{\theta}_2 = -T_2 + \sum_{i=I}^{II} r_{b2}F_n^i + \sum_{i=I}^{II} l_2^i F_f^i \tag{2}$$

where θ_1, θ_2 are the rotation angle of the pinion and the driven gear, respectively; J_1 and J_2 − the mass moments of inertia of the gears; T_1 and T_2 denote the external torques applied on the gear system; r_{b1}, r_{b2} − the base circle radii of the gears; l_1^i and l_2^i − the moment arms.

The dynamic normal load between two meshing gear teeth is expressed as:

$$F_n^i = k_i(t)(r_{b1}\theta_1 - r_{b2}\theta_2 + e_i(t)) + c(r_{b1}\dot{\theta}_1 - r_{b2}\dot{\theta}_2) \tag{3}$$

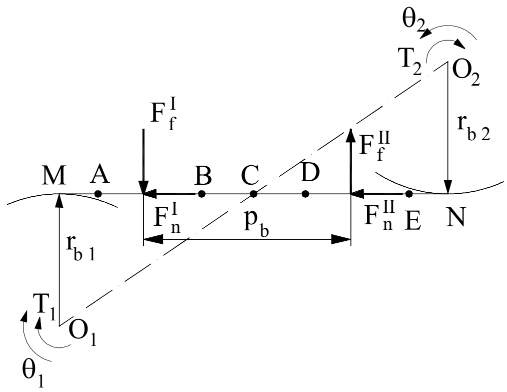

Fig. 1 − Normal and friction loads of the analytical model.

where $k_i(t)$ represents the mesh stiffness of a pair of teeth; c − the damping coefficient; $e_i(t)$ − the equivalent profile error; t − shows the meshing time.

In the paper, the time varying mesh stiffness of a pair of contacting teeth which is calculated by using an accurate analytical model (Atanasiu, 1998).

3. Analytical Models of Friction Coefficient

The calculus procedure of the instantaneous friction coefficient of contacting teeth is based on the Rao's model (Rao, 1979). In the analytical model, the pitch point C is taken as origin and the curve corresponding to the approach or recess path is represented by the following expression

$$\mu = 3\mu_m \frac{y}{l_{a,g}} - 1.5 \cdot \mu_m \left(\frac{y}{l_{a,g}} \right)^2 \tag{4}$$

where y represents the distance between the contact point and the pitch point C; l_a − approach path; l_g − recess path. The analytical relation for determining the average coefficient of friction μ_m in gear teeth (ISO/TR 13989, 2000) can be used especially for spur gear pair with the pitch point in the middle of the path of contact. When $l_a \neq l_g$ it is necessary to consider the specific geometry parameters of the gear pair which is analyzed, where $l_a = AC$ and $l_g = CD$. In this way, the following expression is proposed

$$\mu_m = \frac{\mu_{ma} \cdot l_a + \mu_{mg} \cdot l_g}{l_a + l_g} \tag{5}$$

where μ_{ma}, μ_{mg} are calculated in the middle length of the approach path and of the recess length, respectively.

4. Dynamic Friction Loads

The instantaneous friction forces of the ith meshing pair are derived as follows:

$$F_f^i = \mu F_n^i \tag{6}$$

where μ represents the instantaneous friction coefficient. The calculus procedure of the instantaneous friction coefficient μ of the meshing teeth is developed according to the experimental results (Atanasiu, 2008). The friction force F_f^i changes its sign with the direction of relative sliding velocity, *i.e.*

$$F_f^i = F_f^i \operatorname{sgn}(y) \tag{7}$$

where sgn is the signum function defined as:

$$\operatorname{sgn}(y) = 1, \quad \text{if} \quad V_s > 0 \tag{8}$$

$$\operatorname{sgn}(y) = -1, \quad \text{if} \quad V_s < 0 \tag{9}$$

and V_s represents the sliding speed between mating teeth.

Usually, the distribution factor of the friction load corresponding to a single tooth pair is defined as:

$$c_I = \mu_m \frac{F_n^I}{F_n} \tag{10}$$

In Eq.(6), μ_m is the average friction coefficient, F_n^I represents the normal force corresponded to gear pair $\# I$, and F_n is the total normal force. In the case of a quasi-static rigid model, the total load is shared as $F_n^I = 0.5F_n$ for double-tooth contact and $F_n^I = F_n$ for single-tooth contact.

Under dynamic condition and variable mesh stiffness, the distribution factor c_{dI} of the dynamic friction load of the single tooth pair is proposed as follows:

$$c_{dI} = \mu \frac{F_d^I}{F_n} \tag{11}$$

where μ represents the instantaneous coefficient of friction and F_d^I is the dynamic normal load of the gear pair $\#\mathrm{I}$.

5. Results and Discussions

In this analysis, four different steel gear pairs are considered in order to examine the effects of the addendum modification coefficient on the dynamic friction load s of contacting teeth. Specifications of the pertinent geometrical and kinematics parameters of the analysed gear pairs are shown in Table 1, where m represents the tooth module, z_1, z_2 − number of teeth of the pinion and gear, x_1, x_2 − addendum modification coefficients. These steel spur gear pairs have the face-width of gears, b = 20 mm, and the center distance, a = 90 mm. The segments AB, AC, AD, AE are the geometrical parameters of the analyzed gear pairs. Results presented in Table 1 show that the position of pitch point C on the path of contact AE is dependent to the values of the addendum modification coefficients.

Table 1
Specifications of the Gear Pairs

Gear Pairs	z_1	z_2	m [mm]	x_1	x_2	AB [mm]	AC [mm]	AD [mm]	AE [mm]
GP1	25	47	2.5	−0.3	0.3	5.11	8.12	7.38	12.49
GP2	25	47	2.5	0.8	−0.8	3.74	1.42	7.38	11.12
GP3	18	72	2	0	0	3.96	5.35	5.90	9.86
GP4	18	72	2	0.8	−0.8	2.39	1.15	5.90	8.29

The nominal specific load is $F_n / b =$ 200 N/mm. In the present study, the component of the equivalent profile error is not considered.

A computer program is developed to calculate the numerical values of the instantaneous friction coefficients and dynamic friction loads along the line action in terms of the kinematics characteristics of tooth gears and pinion speed.

The variation of the tooth friction coefficients for a mesh cycle is shown in Figs. 2 *a*−*d*, where μ represents the instantaneous tooth friction coefficient and μ_m is the average friction coefficient for a pinion speed $\omega = 280$ s^{-1}. Fig. 3 presents the variation of the factors c_I and c_{dI} corresponding to static and dynamic friction forces along the engagement line.

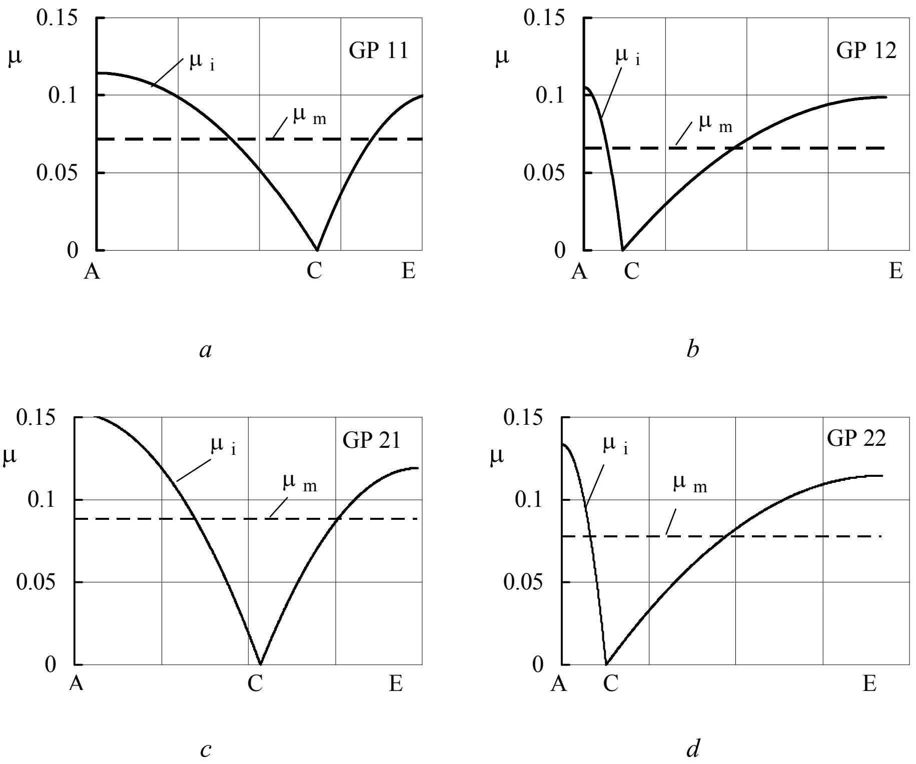

Fig. 2 − Predicted tooth friction coefficients on a mesh cycle.

In these figures, BD represents the single tooth contact and AB and CD are the segments of double tooth contact. The position of the pitch point C on the segments AB or BD is influenced by the values of the addendum modification coefficients.

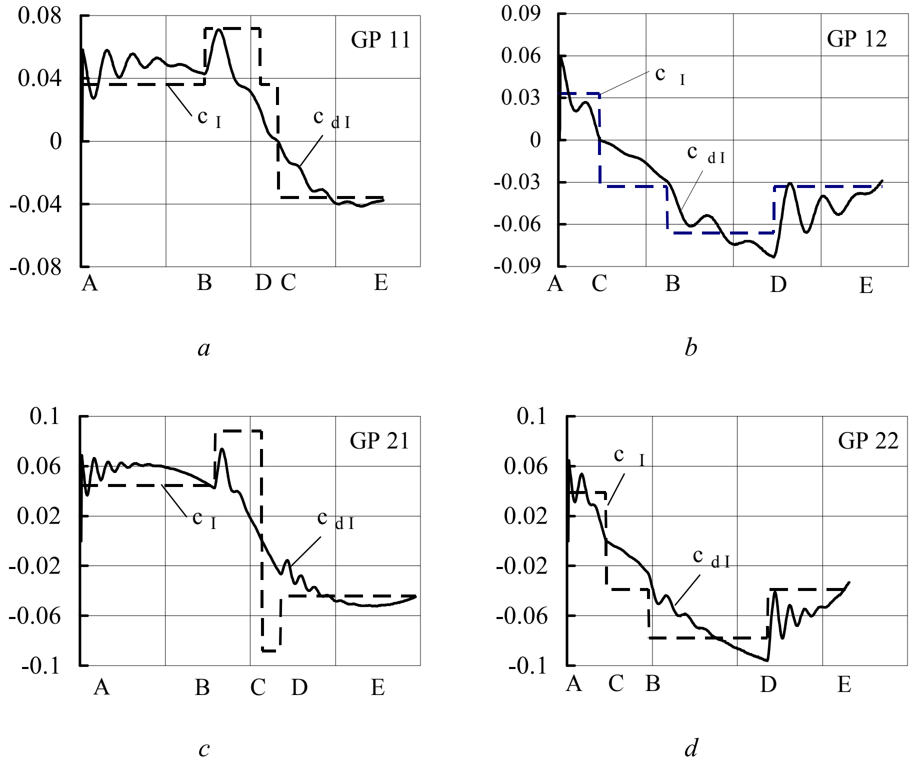

Fig. 3 – Comparison of the distribution factors c_I and c_{dI}.

The analysis of the results presented in Figs. 2 and 3 permits to underline the relevant effect of the addendum modification coefficients on the variation of the instantaneous coefficient of friction and tooth friction loads.

6. Conclusions

The paper presents the methodology developed to evaluate dynamic friction forces of contacting teeth as a function of shared dynamic normal load and instantaneous friction coefficient of contacting teeth under dynamic conditions. The time varying mesh stiffness is included in the dynamic model of the gear pairs. The comparisons of the numerical results of the tooth dynamic friction loads for a mesh cycle are presented for two calculus models. The effects of the addendum modification coefficients on the time-varying sliding friction between contacting teeth and on the dynamic friction loads of gear systems have been investigated.

REFERENCES

* * * ISO/TR 13989, *Calculation of Scuffing Load Capacity of Cylindrical, Bevel and Hypoid Gears* (2000).

Atanasiu V., *An Analytical Investigation of the Time-Varying Mesh Stiffness of Helical Gears.* Bul. Inst. Polit. Iaşi, **XLIV**, *1-2*, V, 7−17 (1998).

Atanasiu V., *An Analytical Investigation of the Shared Friction Loads of Spur Gear Pairs,* Proceedings of the 19th International DAAAM Symposium, 22-25th October 2008, 39−40.

Atanasiu V., *Evaluation of Dynamic Friction Forces of Geared Servomechanisms*. Proceedings of the 20th DAAAM World Symposium, Vienna, 25-28th November 2009, 739−740.

Benedict G.H., *Instantaneous Coefficients of Gear Tooth Friction.* ASLE Transaction, **4**, 57–70 (1961).

Gunda R., *Dynamic Analysis of Sliding Friction in a Gear Pair.* Proceedings of DETC'03, ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Chicago, Illinois USA, September 2-6, 2003, 1−8.

Martin K.F., *A Review of Friction Predictions in Gear Tooth.* Wear, **49**, *2*, 201–238 (1978).

Radzimovsky E.I, *Dynamic Behaviour of Gear Systems and Variation of Coefficient of Friction and Efficiency During the Engagement Cycle.* ASME Journal of Engineering Industry, **97**, *4*, 1247−1281 (1975).

Rao A.C., *Gear Friction Coefficient and Forces.* Wear, **53**, 87−93 (1979).

Rebbechi B., *Measurement of Gear Tooth Dynamic Friction.* DE-Vol.88, Power Transmission and Gearing Conference, ASME, 1996, 355−363.

Vaishya M., Singh R., *Strategies for Modeling Friction in Gear Dynamics.* Journal of Mechanical Design*,* Transaction of the ASME, **125**, 383−393 (2003).

Velex P., *An Analytical Study of Tooth Friction Excitations in Erroless Spur and Helical Gears,* Mechanism and Machine Theory, **37**, 641−658 (2002).

INFLUENŢA DEPLASĂRILOR DE PROFIL ASUPRA FORŢELOR DINAMICE DE FRECARE LA ANGRENAJE CILINDRICE

(Rezumat)

În lucrare se analizează interdependenţa dintre mărimea deplasărilor de profil ale danturii roţilor dinţate şi variaţia forţelor dinamice de frecare ale dinţilor în contact pe parcursul unui ciclu de angrenare. Modelarea analitică pentru forţele dinamice de frecare ale danturii include distribuţia forţei normale corespunzător perechilor de dinţi în angrenare pe baza unui model dinamic care include variaţia rigidităţii danturii şi unei metodologia propusă de autori pentru coeficientul instantaneu de frecare. Rezultatele numerice sunt obţinute prin utilizarea unui program de calcul original care permite analiza procesului de angrenare în regim dinamic, cu evidenţierea influenţei deplasărilor de profil asupra mărimii şi variaţiei coeficientului de frecare instantaneu şi a forţelor dinamice de frecare.