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ON THE "PSEUDO INTELLIGENCE" OF THE SHAPE MEMORY ALLOYS THROUGH THEIR OWN "FRACTAL NEURAL NETWORK"

BY

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Abstract. Assimilating the shape memory alloys as a complex systems, in the frame of Extendend Scale Relativity Theory, *i.e.* a fractal theory of motion with a arbitrary constant fractal dimension, some behaviours of such materials (shape memory effect, pseudo elasticity) are analyzed. In our opinion these behaviours can be associated to a pseudo intelligence of the shape memory alloys given by its "fractal neural network".

Keywords: shape memory alloys; fractal theory; pseudo elasticity.

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1. Introduction

Complex system is a very favorable medium for the appearance of instabilities (Chen, 1994; Morozov, 2012). These instabilities imply both chaos through different routes (intermittencies, quasi-periodicity, cascade of period-doubling bifurcations, sub-harmonic bifurcations, torus breakdown) and self-structuring through generation of complex structures (Dimitriu *et al*., 2007; Agop *et al.*, 2013; Dimitriu *et al*., 2013). According to the classical concepts, all theoretical complex system models (fluid models, kinetic models, etc.) assume that the dynamics of the complex system particles occur on continuous and differentiable curves, so that they can be described in terms of continuous and differentiable motion variables (energy, momentum, density, etc.). These motion variables are exclusively dependent on the spatial coordinates and time. In reality, the complex system dynamics proves to be much more complex and the above simplifications cannot be expected to explain all of the aspects of the complex system dynamics. However, this situation can still be standardized if we consider that the complexity of complex system interaction processes impose different time resolution scales, while the evolution complex system patterns lead to different degrees of freedom.

From the above mentioned arguments it results that the explication of the complex system dynamics can be based on the assumption that the motions of the complex system particles take place on continuous but non-differentiable curves (fractal curves) since, according to the procedures (Nottale, 1993; Nottale, 2011; Munceleanu *et al*., 2011; Agop & Magop, 2012), only a fractal curve is dependent on the scale resolution. Moreover, according to the methodology (Gouyet, 1992), through dynamics of special topologies which can implement evolution patterns in complex system, it can lead to various degrees of freedom. Such an assumption can be sustained by a typical example, related to the collision processes in complex system: between two successive collisions the trajectory of the complex system particle is a straight line that becomes non-differentiable at the impact point. Considering that all the collision impact points form an uncountable set of points, it results that the trajectories of the complex system particles become continuous but non-differentiable curves.

Since the non-differentiability (fractality) appears as a fundamental property of the complex system dynamics, it seems necessary to construct a corresponding non-differentiable complex system physics. We assume that the complexity of interactions in the complex system dynamics is replaced by non-differentiability (fractality). This topic (fractal motion) was systematically developed using either the Scale Relativity Theory (SRT) (Nottale, 1993; Nottale, 2011), or the Extended Scale Relativity Theory (ESRT), *i.e.* the Scale Relativity Theory in an arbitrary constant fractal dimension (Munceleanu *et al*., 2011; Agop & Magop, 2012).

Therefore, as described above, a Euclidian dynamics of a complex system with external constraints is replaced with the fractal dynamics of a complex system free of any external constraints. Practically, the motion with constraints in the Euclidian space, *i.e*. on continuous and differentiable curves, is replaced by a motion free of constraints in the fractal space, *i.e.* on continuous but non-differentiable curves. To do this, we have to admit the correspondence between the austenite-martensite phase transition in the shape memory alloys (Cisse *et al*., 2016; Stanciu, 2009) and the "fractality" of the motion trajectories of the complex system structural units (given by means of the scale resolution and the fractal dimension of the motion curve).

Let us reconsider the fractal hydrodynamic equations with an arbitrary constant fractal dimensions, *i.e.* the specific fractal momentum and fractal state density conservation laws (Munceleanu *et al*., 2011; Agop & Magop, 2012):

$$\partial_t \cdot v^l + v^i \cdot \partial_i \cdot v^l = -\partial^l \left(Q+u\right) \tag{1}$$

$$\partial_t \cdot \rho + \partial_l \cdot \left(\rho v^l\right) = 0 \tag{2}$$

with Q the specific fractal potential

$$Q = -2\lambda^2 \left(dt\right)^{\left(\frac{4}{D_F}\right)-2} \frac{\partial_i \cdot \partial^i \sqrt{\rho}}{\sqrt{\rho}} = \frac{u^l u_l}{2} - \lambda\left(dt\right)^{\left(\frac{2}{D_F}\right)-2} \partial_l \cdot u^l \tag{3}$$

In the previous relations

$$v^l = 2\lambda\left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \partial^l \cdot s \tag{4}$$

is the standard classical speed which is differentiable and independent of the scale resolution *dt*,

$$u^l = \lambda\left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \partial^l \ln\rho \tag{5}$$

is the non-standard speed which is non differentiable and scale resolution dependent, ρ the states density, U the external scalar potential, λ the fractal – non-fractal transition coefficient and D_F the fractal dimension of the motion curves. The speed fields v^l and u^l defines in the fractal space the complex speed field.

$$\tilde{V}^l = -2i\lambda\left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \partial^l \ln\Psi = v^l - iu^l \tag{6}$$

with $\Psi = \sqrt{\rho}e^{is}$ the wave function, $\sqrt{\rho}$ the amplitude and s the phase. We remind that for time scales that prove to be larger if compared with the inverse of the highest Lyapunov exponent, the deterministic trajectories are replaces by a collection of potential paths and that the concept of “definite position” is substitute by that of the probability density. Thus Ψ does not have any physical significance but $|\Psi|^2 \equiv \rho$ has, as a probability density.

2. Fractal Hydrodynamic Equations

We assume that the motions of the complex system structural units take place on continuous but non-differentiable curves (fractal curves).

Any continuous but non-differentiable curve of the complex system particles (complex system fractal curve) is explicitly scale resolution dt dependent, *i.e.* , its length tends to infinity when dt tends to zero.

We mention that, mathematically speaking, a curve is non-differentiable if it satisfied the Lebesgue theorem, *i.e.* its length becomes infinite when the scale resolution goes to zero (Mandelbrot, 1982). Consequently, in this limit, a curve is as zig-zagged as one can imagine. Thus, it exhibits the property of self-similarity in every one of its points, which can be translated into a property of holography (every part reflects the whole) (Gouyet, 1992; Mandelbrot, 1982).

3. Solution of Fractal Hydrodynamic Equations for Fractal Static States

Let us now consider the fractal static state

$$\partial_t = 0, \quad v^l = 2\lambda (dt)^{\left(2/D_F\right)-1} \partial^l s = 0 \tag{7}$$

which implies “coherence states” (the structural units of the shape memory alloys (SMA) as a complex system) are in phase, *i.e.* $s = const$. Then the fractal hydrodynamics equations system (eqs. (1) - (3)) takes the following form:

$$\partial^l (Q+U) = 0, \quad -2\lambda^2 (dt)^{\left(4/D_F\right)-2} \frac{\partial_i \cdot \partial^i \sqrt{\rho}}{\sqrt{\rho}} + U = E \tag{8}$$

where $E = \langle E \rangle$ represents the total specific fractal energy of the fractal static states. The states density conservation law, eq. (2), is identical satisfied.

In order to solve eq. (8) it should be known the functional dependence $U = U(\rho)$. The simplest choice is $U = C_1 \rho$, with C_1 = const. In such condition eq. (2) in the one dimensional case and with the substitutions

$$\xi = \left(\frac{E}{2}\right)^{1/2} \frac{x}{\lambda (dt)^{\left(2/D_F\right)-1}}, \quad \rho^{1/2} = \left(\frac{E}{C_1}\right)^{1/2} u \qquad E > 0 \tag{9}$$

takes the shape of a Ginzburg – Landau equation type, which will be called Fractal Ginzburg Landau equation (FGLE)

$$\partial_{\xi\xi} u = u^3 - u \tag{10}$$

Generally this type of equations always admits the solution $u_c = \mathrm{th}\left[\left(\xi - \xi_0\right) / \sqrt{2}\right]$ (Cristescu, 2008). In the following we will build a new class of solution for the Ginzburg Landau fractal equation, through which u_c will be find as a particular solution. From such a perspective by multiplying both sides of the eq. (10) with $df / d\xi$ and performing the integration over ξ we obtain:

$$\partial_{\xi} u = \left(\frac{1}{2}u^4 - u^2 + C_2\right)^{1/2} \tag{11}$$

where

$$C_2 = \left[\left(\partial_{\xi} u\right)^2 - \frac{1}{2}u^4 + u^2\right]_{\xi=\xi_0} \tag{12}$$

Eq. (12) is obviously a restriction imposted on the order parameter *u*, showing the boundary conditions a further integration of eq. (11) leads to

$$\frac{\left(\xi - \xi_0\right)}{2^{1/2}} = \int_0^u \frac{du}{\left[\left(u_1^2 - u^2\right)\left(u_2^2 - u^2\right)\right]^{1/2}} \tag{13}$$

where

$$u_{1,2}^2 = 1 \mp \left(1 - 2C_2\right)^{1/2} \tag{14}$$

and ξ_0 a new constant of integration.

By the change of variable $w = u / u_1$ the integral (13) becomes:

$$\frac{u_2\left(\xi - \xi_0\right)}{2^{1/2}} = \int_0^w \frac{dw}{\left[\left(1 - w^2\right)\left(1 - k^2 w^2\right)\right]^{1/2}} \tag{15}$$

where

$$k = \frac{u_1}{u_2} \tag{16}$$

Writing u_1 and u_2 in term of *k*:

$$u_1^2 = \frac{2k^2}{1+k^2} \ , \ u_2^2 = \frac{2k^2}{1+k^2} \tag{17}$$

the integral (15) becomes

$$\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} = \int_0^{w_1} \frac{dw}{\left[\left(1-w^2\right)\left(1-k^2w^2\right)\right]^{1/2}} \tag{18}$$

with the superior limit

$$w_1 = \left(\frac{1+k^2}{u}\right)^{1/2} \tag{19}$$

The integral (18) can be solved in terms of Jacobi elliptic function of argument $\left(1+k^2\right)^{-1/2}\left(\xi - \xi_0\right)$ and the modulus k (Bowman, 1955). Indeed, by the version of the integral (18) it results the Jacobi's elliptic function:

$$w = \mathrm{sn}\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] \tag{20}$$

of periods:

$$\omega_1 = 4\int_0^{\pi/2} \frac{d\varphi}{\left[\left(1-k^2\sin^2\varphi\right)\right]^{1/2}} \ , \ \omega_2 = 2i\int_0^{\pi/2} \frac{d\varphi}{\left[\left(1-k'^2\sin^2\varphi\right)\right]^{1/2}} \tag{21}$$

$$k^2 + k'^2 = 1$$

from which the solution is obtained by returning to the previous variable u :

$$u = \left(\frac{2k^2}{1+k^2}\right)^{1/2} \mathrm{sn}\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] \tag{22}$$

4. Conclusions

We can now discuss some conclusions:

i) The fractal state density takes the form:

$$u^2 = \left(\frac{2k^2}{1+k^2}\right)\mathrm{sn}^2\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] = \left(\frac{2k^2}{1+k^2}\right) - \left(\frac{2k^2}{1+k^2}\right)\mathrm{cn}^2\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] \tag{23}$$

while the specific fractal potential:

$$q = -u^{-1}\partial_{\xi\xi}u = \left(1 - u^2\right) \equiv \left(\frac{1-k^2}{1+k^2}\right) + \left(\frac{2k^2}{1+k^2}\right)\mathrm{cn}^2\left[\frac{\xi - \xi_0}{\left(1+k^2\right)^{1/2}} : k\right] \quad (24)$$

This mean that the functionality of a SMA is realized through cnoidals oscillatory modes of either fractal states density, or of the specific fractal potential;

ii) The explicit form of the oscillatory modes can be obtained through the degeneration of elliptic function cn in modulus *k*. Thus for

$$k \to 0\,,\ \ \omega_1 \to \frac{\pi}{2}\ ,\ \ \omega_2 \to i\infty \quad (25)$$

we will have the degeneration

$$\begin{gathered}\mathrm{cn} \to \cos \\ n^2 \to 0 \\ q \to 1\end{gathered} \quad (26)$$

For

$$k \to 1\,,\ \ \omega_1 \to \infty\,,\ \ \omega_2 \to i\pi \quad (27)$$

we will have the degeneration

$$\begin{gathered}\mathrm{cn} \to \mathrm{sech} \\ u^2 \to \mathrm{th}^2\left(\frac{\xi - \xi_0}{2^{1/2}}\right) \\ q \to \mathrm{sech}^2\left(\frac{\xi - \xi_0}{2^{1/2}}\right)\end{gathered} \quad (28)$$

The degeneration (25) with (26) corresponds to harmonic package, while the degeneration (27) with (28) to solitonic package. For $k \equiv 0$ harmonic sequence and for $k \equiv 1$ solitonic sequence are obtained. We note that the degeneration $k \equiv 1$ in solution (22) implies the standard solution u_c;

iii) In the classical theory the cnoidal oscillation modes are associated to a Toda lattice (Cristescu, 2008), *i.e.* a system of coupled nonlinear oscillators. Following the same procedure we will attribute the fractal cnoidal oscillation modes to the Toda type fractal lattice. Then the modulus *k* of the elliptic function cn becomes a measure of the coupling between the fractal oscillators of the Toda type fractal lattice: $k \to 0$ shows the total decoupling of the oscillators while $k \to 1$ shows the total coupling of the same oscillator. The functionality of the Toda type fractal lattice induces in the spectrum of the "fractal phononic field" an acoustic branch and an optical one. Such a dependence could explain why in the SMA materials are presented two phases, one called austenite (A), stable at high temperature and low stress and other one called martensite (M), metastable at low temperature and high stress;

iv) In the standard theory through the mapping of a Toda unidimensional lattice (Cristescu, 2008) it can be reduced to a specific neural network. Following the same procedure but applied to our case, one could find the fractal neural networks having specific functions.

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ASUPRA PSEUDOINTELIGENŢEI ALIAJELOR CU MEMORIA FORMEI DATĂ DE PROPRIA LOR REŢEA NEURONALĂ FRACTALĂ

(Rezumat)

Asimilând aliajele cu memoria formei ca sisteme complexe, în cadrul unei teorii fractale a mişcării în dimensiune fractală arbitrar constantă, sunt analizate câteva comportamente specifice ale acestor aliaje. În opinia noastră aceste comportamente sunt dictate de acea presupusă inteligență a materialelor cu memoria formei (pseudointeligență) asociată propriei lor rețele neuronale fractale.