BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 62 (66), Numărul 1, 2016
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

ON A SPECIAL FRACTAL CELLULAR NEURAL NETWORK BY MEANS OF FRACTAL POTENTIAL AND ITS IMPLICATIONS IN THE BACTERIAL GROWTH PROCESS

BY

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Received: January 11, 2016
Accepted for publication: April 18, 2016

Abstract. Considering that nervous impulse transmission in the brain neuronal network is achieved on continuous but non-differentiable curves (fractal curves), in the scale relativity hydrodynamic variant (with constant arbitrary fractal dimension) a special cellular neural network class at fractal scale is introduced. Thus, this solution with infinite fractal “energy” implies through a fractal potential cnoidal oscillation modes of the fractal states density. These modes can be associated with an one-dimensional Toda fractal lattice and, by mapping, with a fractal cellular neural network. The solution with finite fractal “energy” generated by a spontaneous symmetry breaking mechanism induces elements of a fractal logic as fractal bit, fractal gates etc. The implication of the model in the bacterial growth process is given.

Keywords: fractal potential; fractal kink solution; symmetry breaking; Toda lattice; cellular neural network.

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1. Introduction

In recent papers (Gavriluț *et al.*, 2016; Butuc *et al.*, 2016) it is proved that in the fractal approximation of the motion, one can introduce two new concepts of cellular neural networks, precisely, the differentiable cellular neural network and the non-differentiable cellular neural one. Their coherence assures not only the functional-structural compatibility of any living structure, but also their comunication code.

In this paper we introduce a new class of cellular neural networks, based on the fractal potential. In such context, the spontaneous symmetry breaking will generate patterns. Based on their functionality, the specific communication code will be induced.

2. Results and Discussions

2.1. The Mathematical Model

Let us consider the fractal hydrodynamic equations (Colotin *et al.*, 2009; Timofte *et al.*, 2011). Then for the fractal static states, we have

$$\partial_t = 0, \boldsymbol{V}_D = D\left(dt\right)^{(2/D_F)-1} \nabla \mathrm{S} = 0 \tag{1}$$

In phase coherence S = const. of the structural units (Mitchell, 2009) the fractal hydrodynamic equations become with the substitutions

$$U = E\rho, E = \text{const.} > 0, \rho^{1/2} = f, \mathbf{D} = D\left(dt\right)^{(2/D_F)-1} \tag{2}$$

become

$$\frac{2m_0\mathbf{D}^2}{E}\Delta f = f^3 - f \tag{3}$$

Particularly, in the one-dimensional case $\xi = x(E/2m_0\mathbf{D}^2)^{1/2}$, (3) takes the form:

$$\partial_{\xi\xi} f = f^3 - f \tag{4}$$

One can refer to Timofte *et al.*, 2011 for the significance of the quantities from (1)-(4).

The eq. (4) can be also obtained by means of the fractal variational principle $\delta \int L d\tau = 0$, with $d\tau$ the fractal elementary volume applied to the fractal Lagrangean density:

$$L = \frac{1}{2}(\partial_\xi f)^2 - \wp\left(f\right) \tag{5}$$

with the fractal potential

$$\wp\left(f\right)=\left(\frac{f^4}{4}\right)-\left(\frac{f^2}{2}\right) \tag{6}$$

The eq. $\partial_{\xi\xi}f=0$ has the solutions $f_F^{(1)}=0, f_F^{(2,3)}=\pm 1$. By calculating the second derivative with respect to ξ of the fractal "potential" (6) and substituting the values $f^{(1,2,3)}$ into the result of this differentiation we find $\wp_{\xi\xi}(0)=-1, \wp_{\xi\xi}(\pm 1)=2>0$, *i.e.*, the solution $f_F^{(2,3)}=\pm 1$ is associated with the minimum fractal "energy". Hence, the model under consideration has a double degenerated vacuum fractal state.

From (5) it results both fractal "energy",

$$\varepsilon\left(f\right)=\int_{-\infty}^{\infty}d\xi\frac{1}{2}\left[(\partial_{\xi}f)^2+\wp\left(f\right)\right] \tag{7}$$

and the fractal "energy" relative to the fractal vacuum

$$\varepsilon\left(f\right)-\varepsilon\left(f_F\right)=\int_{-\infty}^{\infty}d\xi\frac{1}{2}\left[\left(\partial_{\xi}f\right)^2+\frac{1}{4}\left(f^2-1\right)^2\right] \tag{8}$$

Since all terms in (8) are positive and in view of the infinite limits of integration, the finiteness of the fractal "energy" implies that at $\xi\to\pm\infty$

$$\partial_{\xi}f=0, \frac{1}{4}\left(f^2-1\right)^2=0 \tag{9}$$

From this, it follows that at $\xi\to\pm\infty$, the function $f(\xi)$ tends to its fractal vacuum value $f_F^{(2,3)}\to\pm 1$. In order to find the explicit form of the solution of (4), we multiply it by $\partial_{\xi}f$ and subsequently over ξ. This yields

$$\frac{1}{2}(\partial_{\xi}f)^2=-\frac{f^2}{2}+\frac{f^4}{4}+\frac{1}{2}f_0 \tag{10}$$

where f_0 is an integration constant. From this, we have:

$$\xi-\xi^0=\int_0^f\frac{df}{\sqrt{\frac{f^4}{4}-\frac{f^2}{2}+\frac{1}{2}f_0}} \tag{11}$$

where ξ^0 is the other integration constant. To this solution it corresponds for an arbitrary f_0, an infinite value of the fractal "energy" $\varepsilon(f)$. To obtain the solution with finite fractal "energy", we make use of the boundary conditions $f_F^{(2,3)} = \pm 1$. From (10) it results that $f_0 = \frac{1}{2}$. Replacing this value of f_0 into (18), the solution $f_k(\xi)$ of the field eq. (10) with a finite fractal "energy" is

$$f_k(\xi) = f\left(\xi - \xi^0\right) = \tanh\left[\frac{1}{\sqrt{2}}\left(\xi - \xi^0\right)\right] \tag{12}$$

This is called the *fractal kink solution*. Combining (8) with the expression $f_F^{(2)} = 1$ and the expression for f_k, we obtain the fractal "energy" of the fractal kink relative to the fractal vacuum:

$$\varepsilon\left(f_k\right) - \varepsilon\left(f_F\right) = \frac{2\sqrt{3}}{3} \quad . \tag{13}$$

The fractal kink solution is obtained by a fractal spontaneous symmetry breaking (the fractal vacuum state is not invariant with respect to the fractal group of transformations which makes invariant eq. (4), while the fractal lagrangean is invariant with respect to the same group). This corresponds to a fractal pattern.

2.2. Fractal Topology and Logic

A fractal topological method can be applied because the admissible number of fractal kinks is determined by the fractal topological properties of the fractal symmetry group of eq. (4). In this context, the following problems must be solved:

i) The number of admissible fractal kink solutions determined by the fractal topological properties of the eq. (4);

ii) *The fractal topological charge.*

The fractal kink solution can be obtained as mapping of a fractal spatial zero-sphere S^0, taken at infinity onto the fractal vacuum manifold of the model (4). The fractal homotopy group for this model is $\Pi_0(Z_0) = Z_2$, *i.e.*, the model gives rise to two solutions: a constant solution and the fractal kink solution. Details on an usual homotopic mapping are given in Jackson, 1992.

The associated fractal topological charge is:

$$q = \frac{1}{2}\int_{-\infty}^{\infty} j(\xi)d\xi = \frac{1}{2}\int_{-\infty}^{\infty} \frac{df}{d\xi}d\xi = \frac{1}{2}\left[f\left(+\infty\right) - f\left(-\infty\right)\right] \tag{14}$$

The fractal vacuum solution (absence of spatial gradients) and the fractal kink solution can be characterized by attributing the $q = 0$ and $q = 1$, respectively (the result is obtained by an adequate normalization f). Since (4) is a fractal Ginzburg-Landau equation type, it follows that $q=0$ and the fractal vacuum solution describes the behavior of the fractal fluid in the absence of self-structuring, *i.e.*, its fractal ground states, while $q=1$ and the fractal kink solution describes the behavior of the fractal fluid in the presence of self-structuring, *i.e.*, the fractal pattern.

Now, one can associate to these values of the fractal topological charge, the fractal bit, that is, a fractal physical system which can exist in two distinct fractal states (an unstructured state or of fractal vacuum and a structured one or of fractal superconductivity). These states are used in order to represent $0(dt)$ and $1(dt)$, that is a single binary fractal digit. The only possible operations (fractal gates) that are compatible with such systems are

the fractal IDENTITY

$$0(dt)\to 0(dt), 1(dt)\to 1(dt). \tag{15}$$

and

the fractal NOT (FNOT)

$$0(dt)\to 1(dt), 1(dt)\to 0(dt) \tag{16}$$

Therefore, unlike the standard bit, the fractal bit is a fractal system with two self-similar and scale independent states.

2.3. Cnoidal Oscillation Modes, Toda Lattices and Cellular Neural Networks by Means of Fractal Potential

Based on the method from Jackson (1992), the solution with infinite fractal energy is obtained by inversion of the elliptic integral (18) and has the expression

$$f=\left(\frac{2s^2}{1+s^2}\right)^{1/2} sn\left[\frac{\xi-\xi^0}{\left(1+s^2\right)^{1/2}};s\right] \tag{17}$$

where sn is Jacobi elliptic function of modulus s (Armitage, 2006).

By the degeneracy of the elliptic function sn with respect to its modulus s one obtains for $s\to 1$ a fractal harmonic waves package and fractal harmonic waves for $s=0$, while for $s\to 1$ is obtained a fractal solitons package and for $s=1$, the fractal kink (12).

In such a frame, the normalized fractal potential in the form

$$\bar{Q}=-\frac{1}{f}\frac{d^2 f}{d\xi^2}=1-f^2=\frac{1-s^2}{1+s^2}+\frac{2s^2}{1+s^2}cn^2\left[\frac{\xi-\xi^0}{\left(1+s^2\right)^{1/2}};s\right] \tag{18}$$

where cn is Jacobi elliptic function of modulus s (Armitage, 2006) specifies the fact that the interactions between fractal fluid entities are achieved through cnoidal oscillation modes of the fractal states density. Since these oscillation modes are associated to an one-dimensional Toda fractal lattice (Toda, 1981), then in its functionality we shall distinguish different regimes induced by those of a fractal "fluid" flow: the fractal non-quasi-autonomous given by fractal harmonic waves and fractal harmonic waves packages, the fractal quasi-autonomous regime given by fractal solitons and fractal soliton packages and the fractal transitory one given by fractal mixtures of fractal harmonic waves package - solitons package type etc. For the standard details see Whitman, 1974.

Now, by mapping, the one-dimensional Toda fractal lattice implies a special fractal cellular neural network (CNN) class with her entire soft, "archeology" etc. For the standard details see Drazin, 1992, Jackson, 1992.

It should be noticed the fact that all above mentioned consequences are coordinated by the fractal potential.

3. Conclusions

The main conclusions of the present paper are the following:

i) Assuming that the nervous impulse transmission through brain's neuronal network is achieved on continuous and non-differentiable curves, the hydrodynamic version of scale relativity in arbitrary constant fractal dimension is used (fractal hydrodynamics);

ii) Assuming that the external scalar potential is proportional with the fractal states density, the one-dimensional solution with finite fractal "energy" in the form of fractal kink is obtained. This solution breaks the fractal vacuum symmetry and generates fractal patterns by means of spontaneous symmetry breaking mechanism (these patterns can be identified with bacteria). Then, the phase coherence of the fractal patterns will produce a self-structuring of the fractal vacuum which is interpreted as a tendancy of the complex system to make structures (patterns);

iii) Since the admissible number of fractal kinks is determined by the fractal topological properties of the symmetry group of eq. (4), a topological fractal method can be applied. Then, some elements of a fractal logic as fractal bit, fractal gates (fractal IDENTITY, fractal NOT) etc. are obtained;

iv) The solution with infinite fractal "energy" is given. Then, by mes of specific fractal potential, the interactions among fractal "fluids" entities are achieved through cnoidal oscillation modes of the fractal states. Since these oscillation modes are associated to a one-dimensional Toda lattice, in its functionality we shall distinguish three various regimes induced by those of a fractal fluid flow: fractal non-quasi-autonomous regime through fractal harmonic waves and fractal harmonic wave packages, the quasi-autonomous regime through fractal soliton and fractal soliton packages and the transitory one through fractal mixtures of harmonic wave package - solitons package type. Now, by mapping, the one-dimensional Toda lattice can be associated with a special fractal cellular neural network class and hence the entire arsenal implied.

Some implications of the model we discussed in the present paper regarding the functionality of biological systems are given in Stoica *et al*., 2015; Duceac *et al*., 2015a; Duceac *et al*., 2015b; Doroftei *et al*., 2016; Nemeş *et al*., 2015; Postolache *et al*., 2016; Ştefan *et al*., 2016.

In the following we give an example on the role communication codes play in the bacterial growth process. “In recent years it has become clear that the production of N-acyl homoserine lactones (N-AHLs) is widespread in Gram-negative bacteria. These molecules act as diffusible chemical communication signals (bacterial pheromones) which regulate diverse physiological processes including bioluminescence, antibiotic production, plasmid conjugal transfer and synthesis of exoenzyme virulence factors in plant and animal pathogens. The paradigm for N-AHL production is in the bioluminescence (lux) phenotype of Photobacterium fischeri (formerly classified as Vibrio fischeri) where the signaling molecule N-(3-oxohexanoyl)-L-homoserine lactone (OHHL) is synthesized by the action of the LuxI protein. OHHL is thought to bind to the LuxR protein, allowing it to act as a positive transcriptional activator in an autoinduction process that physiologically couples cell density (and growth phase) to the expression of the bioluminescence genes. Based on the growing information on LuxI and LuxR homologues in other N-AHL-producing bacterial species such as Erwinia carotovora, Pseudomonas aeruginosa, Yersinia enterocolitica, Agrobacterium tumefaciens and Rhizobium leguminosarum, it seems that analogues of the P. fischeri lux autoinducer sensing system are widely distributed in bacteria. The general physiological function of these simple chemical signaling systems appears to be the modulation of discrete and diverse metabolic processes in concert with cell density. In an evolutionary sense, the elaboration and action of these bacterial pheromones can be viewed as an example of multicellularity in prokaryotic populations” (Salmond *et al.*, 1995).

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ASUPRA UNEI CLASE DE REŢELE CELULARE NEURALE FRACTALE PE BAZA POTENŢIALULUI FRACTAL ŞI IMPLICAŢIILE ACESTEIA ÎN PROCESUL DE CREŞTERE BACTERIALĂ

(Rezumat)

Considerând că în rețeaua neuronală din creier transmiterea impulsului nervos se realizează pe curbe continue, dar nediferențiabile (curbe fractale), introducem în varianta hidrodinamică a relativității de scală (cu dimensiune fractală arbitrară constantă) o clasă specială de rețele celulare neurale la scară fractală. Prin urmare, soluția de „energie" fractală infinită a modelului implică prin potențialul fractal moduri de oscilație cnoidală ale densității stărilor fractale. Aceste moduri pot fi asociate unei rețele fractale unidimensionale Toda şi, prin mapare, unei rețele celulare neurale fractale. Soluția cu „energie" fractală finită generată printr-un mecanism de rupere spontană de simetrie induce elemente de logică fractală cum ar fi, bitul fractal, porțile fractale etc. Se dă un exemplu asupra rolului jucat de codurile de comunicare ce sunt fundamentate prin logica fractală în procesul de creştere celulară.